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1/f Noise from Systems in Thermal Equilibrium

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ABSTRACT

The power spectra of fluctuations in the mean square Johnson noise voltage across small semiconductor and metal films in thermal equilibrium were measured down to 10^{-4}Hz. The spectra have a 1/f-like behavior that matches the resistance fluctuation spectra obtained by passing a current through the samples. These measurements constitute strong evidence that 1/f noise is due to equilibrium resistance fluctuations.

In this Letter we report the observation of a 1/f-like power spectrum for low frequency fluctuations of the mean square Johnson noise voltage across a very small sample of semiconductor or discontinuous metal film in thermal equilibrium. The 1/f spectrum is shown to be due to resistance fluctuations in the sample, and closely matches the resistance fluctuation spectrum obtained by passing a a current through the sample. Our measurements are the first observation of 1/f noise from a system in thermal equilibrium. The fact that 1/f noise had until now been observed only under non-equilibrium steady state conditions led some authors[1,2] to propose non-equilibrium theories for its origin. Our present results, however, together with earlier work showing that a theory based on equilibrium temperature fluctuations quantitatively predicts the magnitude of 1/f noise in continuous metal films[3], superconducting films[4] biased at T_c, and Josephson junctions[5], constitute strong evidence that 1/f noise is an equilibrium effect.

Consider a resistance, R, of total heat capacity C_v shunted by a capacitance, C, and in thermal contact with a reservoir at temperature T_o. The voltage across the capacitor, V(t), represents a single degree of freedom that can exchange energy with the resistor via the charge carriers in the resistor. This exchange takes place on time scales of order $\tau = RC$. In thermal equilibrium the average energy of the capacitor, $\langle E_c \rangle = \frac{1}{2}C\langle V^2 \rangle = \frac{1}{2}k_B T_o$. These voltage fluctations (Johnson noise) are limited to a bandwidth of $1/4\tau$, and consequently have a spectrum of the form $S_V(f) = 4k_BT_oR/[1+4\pi^2f^2\tau^2]$. If the resistor is assumed to exchange energy with the reservoir on a time scale of order τ_R that is much greater than τ, the capacitor is able to reach equilibrium with the internal degrees of freedom of the resistor before the internal energy of the resistor can change. The temperature of the capacitor is then the same as the temperature of the resistor. $V^2(t)$, like $V(t)$, is a rapidly fluctuating quantity in time due to this exchange of energy between the resistor and capacitor. However, the average of $V^2(t)$ over a time, θ, such that $\tau \ll \theta \ll \tau_R$, $\langle V^2(t)\rangle_\theta = k_BT/C$ (T is now the instantaneous temperature of the resistor), is sensitive to slow energy or temperature fluctuations in the resistor on time scales τ_R or longer.

Experimentally, the Johnson noise voltage, $V(t)$, is passed through a filter with a bandpass from f_o to f_1, squared, and averaged over a time $\theta > 1/f_o$ to give $P(t)$, a slowly varying signal proportional to the Johnson noise power in the bandwidth f_o to f_1. Thus,

$$P(t) \approx 4k_BTR\int_{f_o}^{f_1} df/(1+4\pi^2f^2\tau^2) + P_o(t), \tag{1}$$

where $P_o(t)$ represents the fluctuations in $P(t)$ due to the rapid exchange of energy between capacitor and resistor. Because this exchange is so rapid, $P_o(t)$ has a spectrum, $S_{P_o}(f)$, that is independent of f for the low frequencies in which we are interested. S_{P_o} may be reduced by increasing the bandwidth or by moving the bandwidth to higher frequencies, but in practice $P_o(t)$ severly limits the accuracy of measurements of $P(t)$.

If the bandwidth in Eq. (1) is either totally above or totally below the knee at $1/2\pi\tau$, $P(t)$ is sensitive to slow resistance as well as temperature fluctuations. These resistance fluctuations, ΔR, may either be driven by temperature fluctuations with a spectrum $S_T(f)$ so that $\Delta R = \bar{R}\beta\Delta T$, where $\beta \equiv (1/R)\partial R/\partial T$; or be temperature independent fluctuations, ΔR_o, with a spectrum $S_{R_o}(f)$ (such as number or mobility fluctuations of the charge carriers). Thus, from Eq. (1), $\Delta P(t)/\bar{P} = (1 \pm \beta T_o)\Delta T/T_o + \Delta R_o/\bar{R} + P_o(t)/\bar{P}$, and the relative power spectrum for fluctuations in $P(t)$ is of the form

$$\frac{S_P(f)}{\bar{P}^2} = (1 \pm \beta T_o)^2 \frac{S_T(f)}{T_o^2} + \frac{S_{R_o}(f)}{\bar{R}^2} + \frac{S_{P_o}}{\bar{P}^2}, \tag{2}$$

where the plus sign corresponds to $f_o < f_1 \ll 1/2\pi\tau$, and the minus sign corresponds to $f_1 > f_o \gg 1/2\pi\tau$. If, however, most of the noise power and the knee frequency, $1/2\pi\tau$, are included in the bandwidth (i.e. $f_o \ll 1/2\pi\tau \ll f_1$), from Eq. (1) we find $P(t) \approx k_B T/C + P_o(t)$ and $S_P(f)/\bar{P}^2 = S_T(f)/T_o^2 + S_{P_o}/\bar{P}^2$. In this limit, $P(t)$ is not sensitive to resistance fluctuations. Thus, with an appropriate choice of bandwidth, the low frequency spectrum of $P(t)$ is an equilibrium measurement of $S_T(f)$ or $S_{R_o}(f)$ provided the temperature or resistance fluctuations are large enough to dominate S_{P_o}.

Our initial measurements were on evaporated InSb films with a thickness of 1000Å and a resistivity of about 1Ωcm. Previous measurements on similar InSb films[3] had shown that the 1/f noise did not arise primarily from temperature-induced fluctuations: the noise was too large in mangitude, and spatially uncorrelated. Therefore, we expected to observe only the resistance fluctuations $\Delta R_o(t)$. In order to make the relative resistance fluctuation spectrum, $S_{R_o}(f)/\overline{R}^2$, large enough to dominate $S_{P_o}/\overline{P}^2$, the samples were made as small as possible. The resistance of a strip of InSb was monitored while the strip was cut transversely with a diamond knife until only a small bridge containing typically about 10^6 atoms remained. In the presence of a direct current, I, there are voltage fluctuations across the sample, $\Delta V(t) = I\Delta R_o(t)$, which have a relative spectrum $S(f) = S_V(f)/\overline{V}^2 = S_{R_o}(f)/\overline{R}^2$, where $\overline{V}$ is the average voltage across the sample. The solid line in Fig. (1) shows S(f) for a 20MΩ bridge of InSb measured with a direct current. The noise voltage was amplified, digitized, and analyzed by a PDP-11 computer using a Fast Fourier Transform algorithm to determine the power spectrum. The spectrum was remeasured using an ac technique in which a square wave current was applied to the sample, and the PDP-11 was used as a digital lock-in detector to measure the spectrum of the amplitude fluctuations of the induced voltage. The relative spectrum is plotted with open circles in Fig. (1). In a third technique the current was supplied as a series of pulses to reduce the power dissipated in the sample. The relative spectrum is shown in Fig. (1) as open triangles. All three techniques measure the resistance spectrum, $S_{R_o}(f)/\overline{R}^2$. The agreement of the three spectra demonstrates that neither a direct current nor a constant dissipation of power is the cause of the 1/f spectrum.

For the measurement of P(t), the sample was capacitively coupled to a preamplifier to prevent any leakage current flowing through the sample. The input capacitance produced a knee frequency, $1/2\pi RC \approx 500$Hz, in the Johnson noise spectrum. After amplification the noise was filtered with a 10kHz to 300kHz bandpass filter, squared with an analog multiplier, and filtered to remove frequencies above the digitizing frequency. Since the bandpass is above the knee frequency the calculated relative spectrum of this signal is given by Eq. (2) (with the minus sign), while the measured relative spectrum is shown as the open squares in Fig. (1). The white spectrum above 1Hz represents $S_{P_o}/\overline{P}^2$. The 1/f spectrum below 1Hz closely matches the current-biased measurements. To insure that the 1/f spectrum was generated by fluctuations in the sample rather than by spurious effects from our electronics, the InSb was replaced by a metal film resistor (which did not exhibit 1/f noise) of the same resistance. This relative spectrum is shown dotted in Fig. (1). The spectrum is white down to the lowest frequency measured, and represents only the term $S_{P_o}/\overline{P}^2$.

We have made similar measurements on metal films. Although the three current-biased techniques give identical relative spectra for continuous metal films in which the resistance fluctuations are temperature induced[3], we were unable to make these films small enough for $S_R(f)/\overline{R}^2$ to dominate $S_{P_o}/\overline{P}^2$ at frequencies down to 10^{-3}Hz. Very thin (~100Å) metal films, however,(in which current transport is probably partly via metallic conduction, and partly via a hopping process) exhibit much greater noise[6] than can be accounted for by temperature induced resistance fluctuations.

In Fig. (2), the continuous curve is the relative spectrum of a very thin Nb film ($R \approx 200k\Omega$) measured with an ac current bias. The open squares are a Johnson noise measurement with a bandwidth of 100kHz to 200kHz, above the knee frequency of 40kHz. The agreement below 10^{-2}Hz is excellent. The dotted spectrum was obtained from the same sample using a bandwidth of 5kHz to 200kHz, which includes the knee frequency and most of the Johnson noise power. Although the low frequency spectrum is substantially reduced (as expected when P(t) is no longer sensitive to resistance fluctuations), it is still above the background spectrum of a large metal film resistor. This residual noise is possibly due to the temperature fluctuation term $S_T(f)/T_o^2$. Indeed, the assumption[3] of a 1/f spectrum for $S_T(f)$ for a sample of 10^6 atoms yields $S_T(f)/T_o^2 \sim 3\times10^{-7}/f\ Hz^{-1}$; a value that is consistent with the observed spectrum.

Our results strongly suggest that 1/f noise in semiconductors and discontinuous metal films arises from equilibrium resistance fluctuations. Current-biased measurements probe these resistance fluctuations, but in no way generate them. This idea is consistent with several current theories of 1/f noise that propose various mechanisms for the resistance fluctuations, for example: surface traps modulating the number of carriers in semiconductors[7]; mobility fluctuations of the carriers in semiconductors and ionic solutions[8,9]; and temperature induced resistance fluctuations in metal films[3], superconducting films at the transition[4], and Josephson junctions[5]. Our equilibrium measurements are, however, obviously inconsistent with theories that rely on non-equilibrium processes, for example: turbulence theories[1]; theories that require a long term steady current or power[2]; and theories involving thermal feedback via the heat generated by an external current.

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FIGURE CAPTIONS

Fig. 1. InSb bridge: $S_V(f)/\overline{V}^2$ using dc bias (—), ac bias (o), pulsed current bias (Δ); Johnson noise measurement, $S_P(f)/\overline{P}^2$(□). Background $S_P(f)/\overline{P}^2$ from metal film resistor (...).

Fig. 2. Nb bridge: $S_V(f)/\overline{V}^2$ using ac bias (—); $S_P(f)/\overline{P}^2$(□). $S_P(f)/\overline{P}$ including knee frequency (···).

0 0 0 0 4 3 0 6 2 4 4

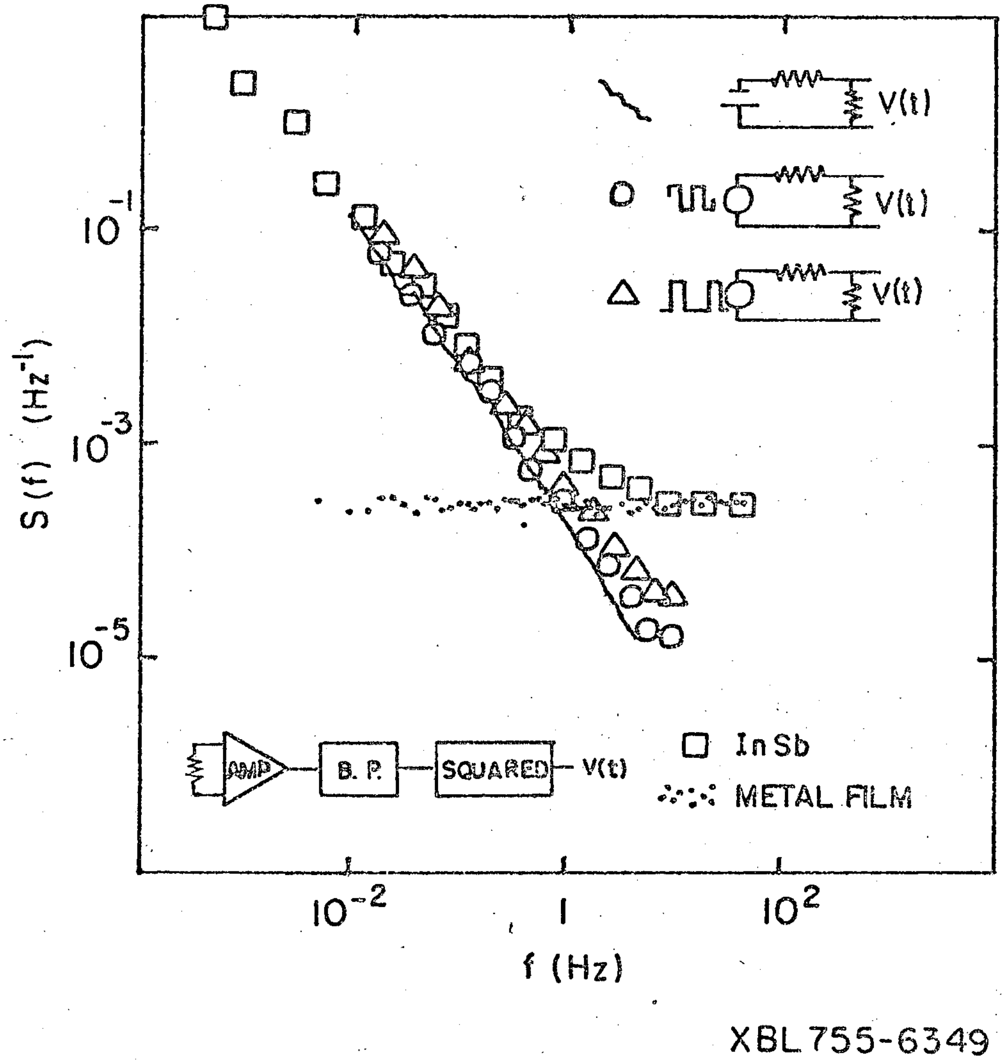

Fig. 1

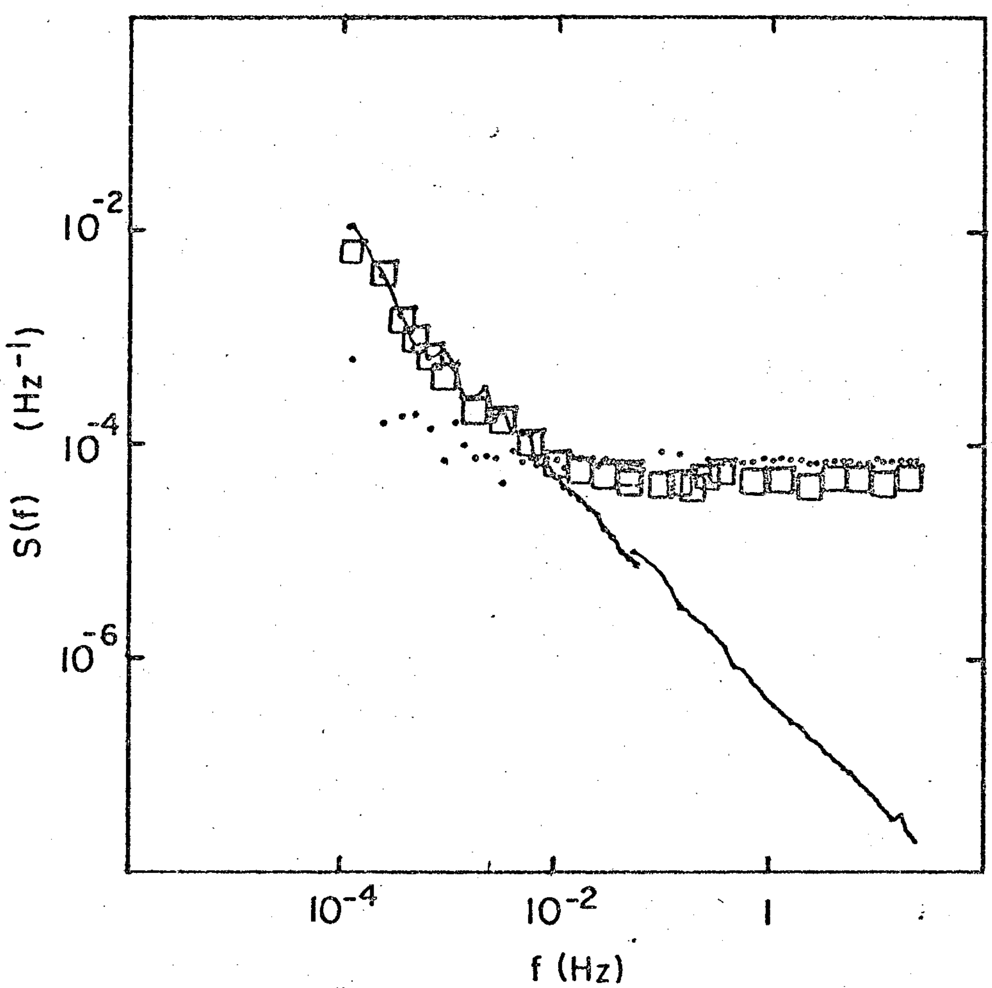

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Fig. 2